SCRIPT `MOD3S1B`: LIMITING DISTRIBUTIONS

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1. Convergence of a t-variate with $n-1$ D.o.F.

We will take repeated draws of samples from a standard normal. For each sample, we compute the t-variate as $t = \frac{\bar{x}}{\sqrt{s^2}/\sqrt{n}}$, where $s^2 = (1/n)\sum_{i=1}^{n}(x_i - \bar{x})^2$. Clearly, this t-variate as a sequence random variable, as it depends on the sample size n. In fact, a t statistic based on an underlying sample size of n is said to have $n-1$ degrees of freedom (D.o.F.). We will see that the sampling distribution of this t-variate converges to the standard normal under increasing sample size. This is referred to as *convergence in distribution*.

```
R> R<-100000 #number of repeated draws
R> mu<-0
R> sig<-1
R> t5<-rep(0,R)  #will collect t-draws with DoF 5-1
R> t10<-rep(0,R) #will collect t-draws with DoF 10-1
R> t100<-rep(0,R) #will collect t-draws with DoF 100-1
R> for (i in 1:R){
 int<-rnorm(5,mu,sig)
 m<-mean(int)
 std<-sd(int)
 t5[i]<-m/(std/sqrt(5))

 int<-rnorm(10,mu,sig)
 m<-mean(int)
 std<-sd(int)
 t10[i]<-m/(std/sqrt(10))

 int<-rnorm(100,mu,sig)
 m<-mean(int)
 std<-sd(int)
 t100[i]<-m/(std/sqrt(100))

 }
R> # standard normal for comparison
R> int<-rnorm(R,mu,sig)
```

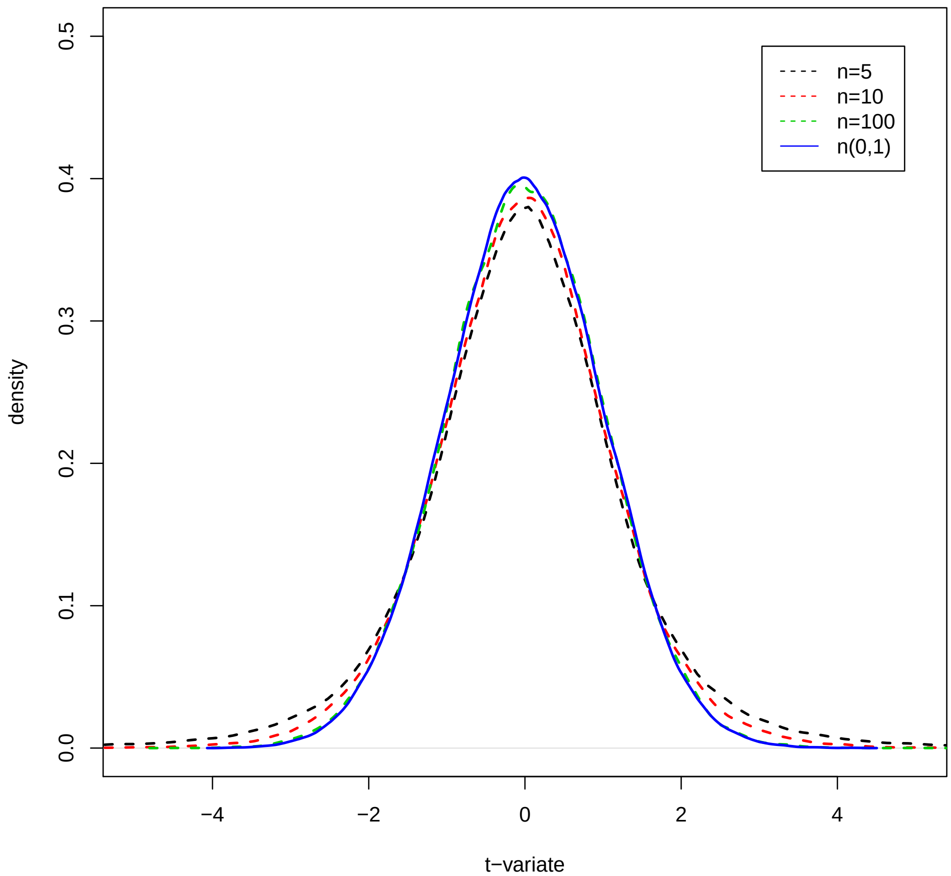

FIGURE 1. Convergence of $t\,(n-1)$ to a $n\,(0,1)$

2. Convergence of a squared t-variate with $n-1$ D.o.F.

According to our lecture notes, a squared t variate with mean zero and variance $(n-1)/(n-3)$ should converge to a χ^2 distribution with one degree of freedom. Let's see if this is true.

```
R> R<-100000 #number of repeated draws
R> mu<-0
R> sig<-1
R> t5<-rep(0,R)  #will collect t-draws with DoF 3-1
R> t10<-rep(0,R) #will collect t-draws with DoF 10-1
R> t100<-rep(0,R) #will collect t-draws with DoF 100-1
R> for (i in 1:R){
 int<-rnorm(5,mu,sig)
 m<-mean(int)
 std<-sd(int)
 t5[i]<-(m/(std/sqrt(5)))^2
```

```
 int<-rnorm(10,mu,sig)
 m<-mean(int)
 std<-sd(int)
 t10[i]<-(m/(std/sqrt(10)))^2

 int<-rnorm(100,mu,sig)
 m<-mean(int)
 std<-sd(int)
 t100[i]<-(m/(std/sqrt(100)))^2

 }
R> # draws from the Chi-2 with 1 d.o.f., for comparison
R> int<-rchisq(R,1)
R>
```

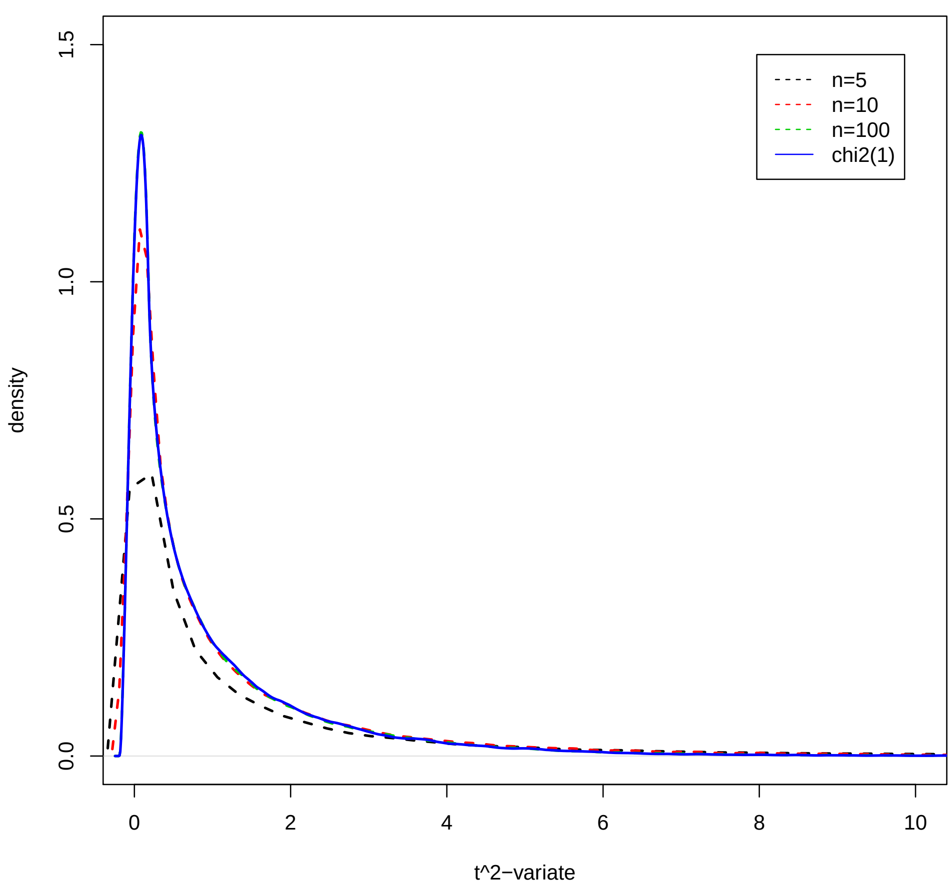

FIGURE 2. Convergence of $(t\,(n-1))^2$ to a $\chi^2\,(1)$

```
R> proc.time()-tic
   user  system elapsed
  33.75    0.08   33.85
```