SCRIPT MOD3S1C: STABILIZING TRANSFORMATION OF THE SAMPLE MEAN

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1. Normal sampling

We will show that while the sampling distribution of $\bar{x}$ converges in distribution to the population mean μ, the distribution of the transform $\sqrt{n}\left(\bar{x}-\mu\right)/\sigma$ converges to a standard normal density. Since this property holds for *any* underlying density of x_i, we will first illustrate it for x_i drawn from a normal, then from a uniform density.

```
R> mu<-3 #true population mean
R> sig<-2 #population std
R> R<-10000 #number of repeated draws
R> m10<-rep(0,R)  #will collect means for samples of size 10
R> m100<-rep(0,R) #will collect means for samples of size 100
R> m1000<-rep(0,R) #will collect means for samples of size 1000
R> for (i in 1:R){
 int<- rnorm(10,mu,sig)
 m10[i]<-sqrt(10)*((mean(int)-mu)/sig)

 int<- rnorm(100,mu,sig)
 m100[i]<-sqrt(100)*((mean(int)-mu)/sig)

 int<- rnorm(1000,mu,sig)
 m1000[i]<-sqrt(1000)*((mean(int)-mu)/sig)
 }
R> # standard normal for comparison
R> snorm<-rnorm(100000,0,1)
R>
```

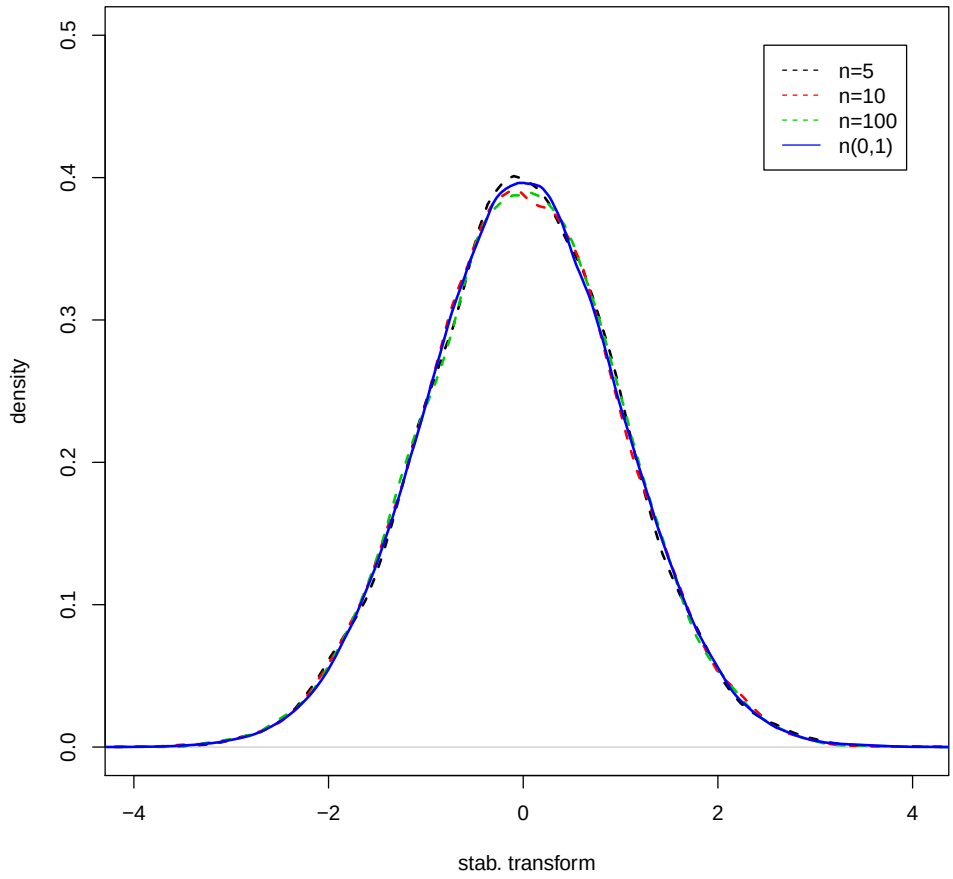

FIGURE 1. Sampling distribution for the stabilized transform of the sample mean under increasing sample size, with x_i drawn from a normal density

2. UNIFORM SAMPLING

```
R> a<--3
R> b<-9
R> mu<-(a+b)/2
R> sig<-sqrt((b-a)^2/12)
R> #this will yield uniform draws with mean 3 and std sqrt(12)
R> R<-10000 #number of repeated draws
R> m10<-rep(0,R)  #will collect means for samples of size 10
R> m100<-rep(0,R) #will collect means for samples of size 100
R> m1000<-rep(0,R) #will collect means for samples of size 1000
R> for (i in 1:R){
 int<- matrix(runif(10,a,b),10)
 m10[i]<-sqrt(10)*((mean(int)-mu)/sig)
```

```
int<- matrix(runif(100,a,b),100)
m100[i]<-sqrt(100)*((mean(int)-mu)/sig)

int<- matrix(runif(1000,a,b),1000)
m1000[i]<-sqrt(1000)*((mean(int)-mu)/sig)
}
```

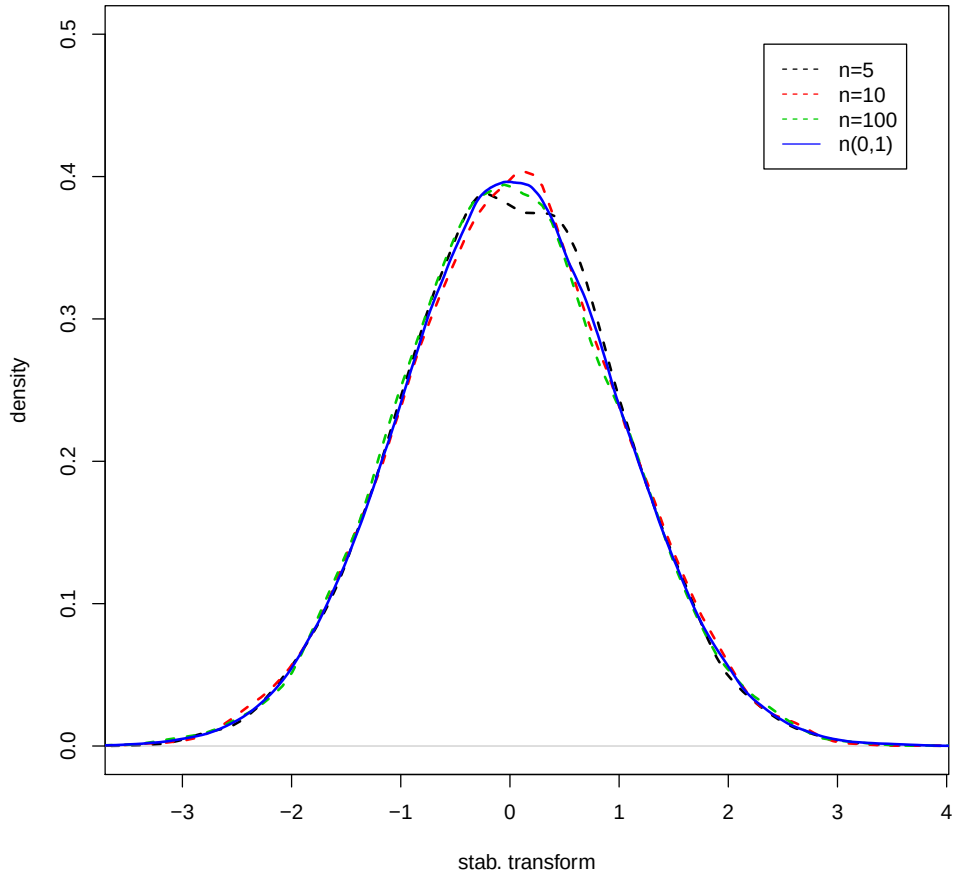

FIGURE 2. Sampling distribution for the stabilized transform of the sample mean under increasing sample size, with x_i drawn from a uniform density

```
R> proc.time()-tic
   user  system elapsed
   3.29    0.03    3.37
```