

## SCRIPT MOD4S2D: MULTIPLICATIVE HETEROSKEDASTICITY

INSTRUCTOR: KLAUS MOELTNER

### LOAD AND PREPARE DATA

This example uses Greene's credit card expenditure data from Example 8.1, p. 159 (6<sup>th</sup> edition). The case with multiplicative HSK is discussed on p. 170 - 172. We will focus on the FGLS solution. Note: One shortcoming of this data set is its small sample size of 72 observations. Since the HSK test results, FGLS, and robust OLS rest entirely on asymptotic results, we need to interpret the output from this exercise with caution.

```
R> data<- read.table('c:/Klaus/AAEC5126/R/data/creditcard.txt', sep="\t", header=FALSE)
R> #
R> #assign variable names
R> names(data)[1]<-"monthexp"
R> names(data)[2]<-"age"
R> names(data)[3]<-"income"
R> names(data)[4]<-"ownhome"
R> save(data, file = "c:/Klaus/AAEC5126/R/data/creditcard.rda")
R> attach(data)
```

Variable definitions:

```
% Contents of Data (columns)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% 1   monthexp   monthly credit card expensiture (1992 dollars)
% 2   age        age (years)
% 3   income     annual income (in $10,000's)
% 4   ownhome    1 = person owns a home
```

### GENERIC OLS AND RESIDUAL PLOTS

```
R> y<- monthexp
R> n<-length(y)
R> X<-cbind(rep(1,n), age, income, income^2, ownhome)
R> k<-ncol(X)
R> bols<-solve((t(X)) %*% X) %*% (t(X) %*% y)
R> e<-y-X%*%bols
R> yhat<-X%*%bols
R> SSR<-(t(e)%*%e)
R> s2<-(t(e)%*%e)/(n-k)
R> Vb<-s2[1,1]*solve((t(X))%*%X)
R> se=sqrt(diag(Vb))
R> tval=bols/se
R> #
```

```
R> ttols<-data.frame(col1=c("constant","age","income","income^2","ownhome"),
  col2=bols,
  col3=se,
  col4=tval)
R> colnames(ttols)<-c("variable","estimate","s.e.,""t")
```

TABLE 1. OLS Results

variable	estimate	s.e.	t
constant	-237.147	199.352	-1.190
age	-3.082	5.515	-0.559
income	234.347	80.366	2.916
income <sup>2</sup>	-14.997	7.469	-2.008
ownhome	27.941	82.922	0.337

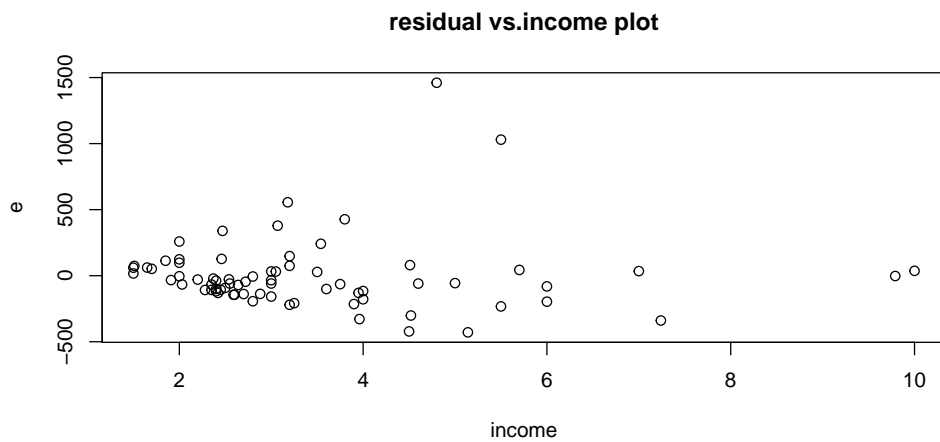
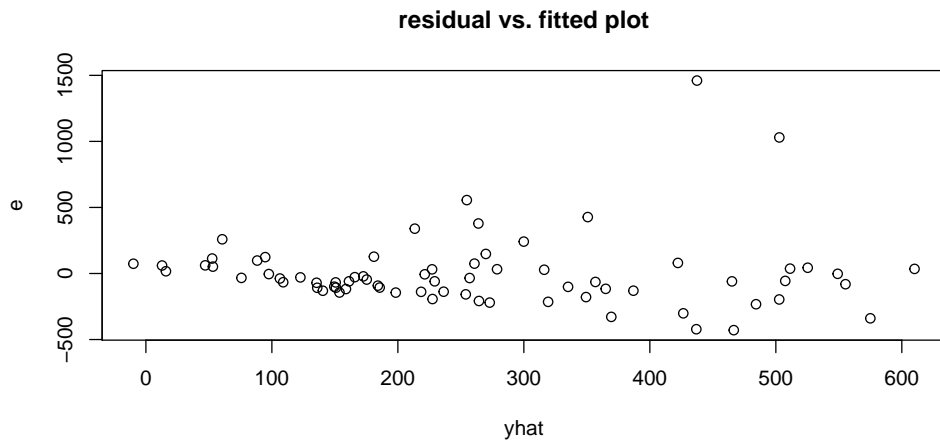


FIGURE 1. HSK diagnostic plots

### BREUSCH-PAGAN TEST

```
R> int<-(t(e)%*% e)/n
R> g<-(e^2/(int[1,1]))-1
R> #capture variables you think may be related to HSK
R> Z<-cbind(rep(1,n), income, income^2)
R> kz<-ncol(Z)
R> LM<-(1/2)*(t(g) %*% Z %*% solve(t(Z) %*% Z) %*% t(Z) %*% g)
R> pval=1-pchisq(LM,kz-1)
```

The BP-statistic for this test is 41.92. The degrees of freedom for the test are 2. The corresponding p-value is 0.

### ROBUST BREUSCH-PAGAN TEST (KOENKER, 1981)

```
R> int<-(t(e)%*% e)/n
R> V<-mean((e^2-(int[1,1]))^2)
R> LM<-(1/V)*
  (t(e^2-int[1,1]) %*% Z %*% solve(t(Z) %*% Z) %*% t(Z) %*% (e^2-int[1,1]))
R> pval=1-pchisq(LM,kz-1)
```

The robust BP-statistic for this test is 6.187. The degrees of freedom for the test are 2. The corresponding p-value is 0.045.

### FGLS

```
R> #Step 1: Consistent estimate of Omega
R> yaux<-log(e^2)
R> Xaux<-cbind(rep(n,1), income, income^2)
R> kaux<-ncol(Xaux)
R> baux<-solve((t(Xaux)) %*% Xaux) %*% (t(Xaux) %*% yaux)
R> sigvec<-as.vector(exp(Xaux %*% baux)+1.2704) #add Harvey's suggested correction
R> Om<-diag(sigvec)
R> #
R> #Step 2: GLS
R> bgls<-solve((t(X)) %*% solve(Om) %*% X) %*% (t(X) %*% solve(Om) %*% y)
R> e<-y-X%*%bgls
R> Vb<-solve((t(X))%*% solve(Om) %*% X)
R> se=sqrt(diag(Vb))
R> tval=bgls/se
R> #
R> ttgls<-data.frame(col1=c("constant","age","income","income^2","ownhome"),
  col2=bgls,
  col3=se,
  col4=tval)
R> colnames(ttgls)<-c("variable","estimate","s.e.,""t")
```

TABLE 2. FGLS Results

variable	estimate	s.e.	t
constant	-117.778	50.520	-2.331
age	-1.242	1.274	-0.975
income	145.373	23.104	6.292
income <sup>2</sup>	-7.944	1.862	-4.266
ownhome	50.998	26.312	1.938

## ROBUST OLS (FOR COMPARISON)

```
R> bols<-solve((t(X)) %*% X) %*% (t(X) %*% y)
R> e<-as.vector(y-X%*%bols)
R> # The "as.vector" coercion is apparently needed for the following diag command
R> # to work properly
R> S<-diag(e^2)
R> Vb<-solve((t(X))%*%X) %*% t(X) %*% S %*% X %*% solve((t(X))%*%X)
R> se=sqrt(diag(Vb))
R> tval=bols/se
R> #
R> ttols<-data.frame(col1=c("constant", "age", "income", "income^2", "ownhome"),
                    col2=bols,
                    col3=se,
                    col4=tval)
R> colnames(ttols)<-c("variable", "estimate", "s.e.", "t")
```

TABLE 3. Robust OLS Results

variable	estimate	s.e.	t
constant	-237.147	212.991	-1.113
age	-3.082	3.302	-0.933
income	234.347	88.866	2.637
income <sup>2</sup>	-14.997	6.945	-2.160
ownhome	27.941	92.188	0.303

```
R> proc.time()-tic
user system elapsed
0.14 0.05 0.20
```