SCRIPT MOD6S2C: NORMAL REGRESSION WITH CONJUGATE PRIORS: CONSTRAINED MODEL

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Simulate data

```
R> n<-10000 #sample size
R> x1<-rep(1,n)
R> x2<-rnorm(n,-1.4,1)
R> x3<-rnorm(n,3,2)
R> btrue<-c(1.2,0.4,0.8)
R> Xtrue<-cbind(x1,x2,x3)
R> X<-cbind(x1,x2)  #KEY: drop last variable for the X used in estimation
R> k<-ncol(X)
R> sig2true<-1.44
R> eps<-rnorm(n,0,sqrt(sig2true))
R> y<-Xtrue %*% btrue + eps
```

Estimation

Define priors, derive posteriors, and compute the log-marginal likelihood:

```
R> #PRIORS:
R> ####################
R> #for beta:
R> mu0<-rep(0,k)
R> V0<-100*diag(k)
R> #for h:
R> v0<-10
R> tau0<-1
R> # get OLS results for convenient expressions of posterior
R> bols<-solve(t(X) %*% X) %*% (t(X) %*% y)
R> e<-y-X%*%bols
R> SSE<-(t(e)%*%e) #previously called "SSR"
R> #
R> # POSTERIORS:
R> ###################
R> V1<-solve(solve(V0)+ t(X) %*% X) # conditional posterior variance of beta
R> mu1<-V1 %*% (solve(V0) %*% mu0 + t(X) %*% y)# conditional posterior mean of beta
R> # last term is equivalent to X'X*bols
R> v1<-v0+n # posterior D.o.F. of h
R> tau1<-v1/(v0*(1/tau0)+SSE+t(bols-as.matrix(mu0))%*% solve(V0+solve(t(X) %*% X))
 %*%(bols-as.matrix(mu0)))
R> #
R> # log-p(y)
R> ###################
```

```
R> logpy<-lgamma(.5*v1) + .5*v0*log(v0*(1/tau0))- lgamma(.5*v0) -.5*n*log(pi)+
 .5*log(det(V1)/det(V0)) - .5*v1*log(v1*(1/tau1))
R> #
R> ttpost<-data.frame(col1=c("constant","x2","h"),
                col2=c(btrue[1:2],1/sig2true),
                col3=c(mu1,tau1),
                col4=c(sqrt(2*tau1^2/v1),sqrt(diag((1/tau1[1,1])*V1))))
R> colnames(ttpost)<-c("variable","true values","post.mean","post.std.")
```

TABLE 1. Posterior results

variable	true values	post.mean	post.std.
constant	1.200	3.562	0.004
x2	0.400	0.383	0.034
h	0.694	0.253	0.020

The log-marginal likelihood value is -21092.675.

Save results for later:

```
R> save(mu1,V1,tau1,v1,logpy,v0,tau0,mu0,V0,
 file = "c:/Klaus/AAEC5126/module6/NormalConjM2.rda")

R> proc.time()-tic
   user  system elapsed
   0.19    0.06    0.25
```