

## SCRIPT MOD3S1D: DELTA METHOD AND KRINSKY-ROBB SIMULATION

INSTRUCTOR: KLAUS MOELTNER

### 1. INTRODUCTION AND DATA

This is an example for deriving the estimated asymptotic variance and standard error for a nonlinear function of original MLE estimates using the DELTA method and Krinsky-Robb simulation.

We will build on the results from `module2s1b`. There, MLE produced a point estimate and the standard error for the standard deviation of the regression error,  $\sigma$ . Here we are interested in the estimate and standard error for the variance  $\sigma^2$ . See our lecture notes for the analytical details.

First, we will load the estimation results from the MLE model with analytical gradient and Hessian, using the wage data.

```
R> load("c:/Klaus/AAEC5126/R/data/wage1000.rda")#load data
R> load("c:/Klaus/AAEC5126/R/data/mod2s1b.rda") #load MLE results
R> names(data) [1]<-"wage"
R> names(data) [2]<-"gender"
R> names(data) [3]<-"race"
R> names(data) [4]<-"unionmember"
R> names(data) [5]<-"education"
R> names(data) [6]<-"experience"
R> names(data) [7]<-"age"
R> attach(data)
R> n<-nrow(data)
R> X<-cbind(rep(1,n),gender,race,unionmember,education,experience)
R> k<-ncol(X)
R> y<-matrix(log(wage))
```

### 2. INFERENCE VIA DELTA METHOD

```
R> sig<-b[k+1]
R> H1<-solve(-H)
R> Vsig<-H1[k+1,k+1]
R> sig2<-sig^2
R> Vsig2<-4*sig^2*Vsig
R> sesig2<-sqrt(Vsig2)
R> #Confidence interval:
R> lo<-sig2-1.96*sesig2
R> hi<-sig2+1.96*sesig2
R> ttDELTA<-data.frame(col1="error variance",
                       col2=sig2,
                       col3=sesig2,
```

```

      col4=lo,
      col5=hi)
R> colnames(ttDELTA)<-c("variable","estimate","s.e.,""lower","upper")

R> ttDELTAx<- xtable(ttDELTA,
  caption="MLE results for error variance using DELTA method")
R> digits(ttDELTAx)<-3 #decimals to be shown for each column
R> print(ttDELTAx,include.rownames=FALSE,
  latex.environment="center", caption.placement="top",table.placement="!h")

```

TABLE 1. MLE results for error variance using DELTA method

variable	estimate	s.e.	lower	upper
error variance	0.224	0.009	0.205	0.243

### 3. INFERENCE VIA KRINSKY-ROBB METHOD

The MLE estimate for  $\sigma$ , call it  $\hat{\sigma}$ , will be asymptotically normally distributed, with mean  $\sigma$ , and variance the  $(k+1)^{th}$  element of the inverted information matrix. We'll approximate the former by the estimate itself, and the latter by the  $(k+1)^{th}$  element of the inverted negative Hessian.

We then take  $R$  draws from this asymptotic distribution. We square each draw to obtain the asymptotic distribution of  $\sigma^2$ .

```

R> R<-10000; #number of repetitions
R> #Step 1: Draw R sig's from empirical density
R> sigvec<-rnorm(R,sig,sqrt(Vsig))
R> #Step2: For each draw, compute function of interest
R> sig2vec<-sigvec^2
R> #Examine statistics of interest
R> sig2<-mean(sig2vec)
R> sesig2<-sd(sig2vec)
R> lo<-quantile(sig2vec,0.025)
R> hi<-quantile(sig2vec,0.975)
R> ttKR<-data.frame(col1="error variance",
  col2=sig2,
  col3=sesig2,
  col4=lo,
  col5=hi)
R> colnames(ttKR)<-c("variable","estimate","s.e.,""lower","upper")

R> ttKRx<- xtable(ttKR,
  caption="MLE results for error variance using Krinsky-Robb method")
R> digits(ttKRx)<-3 #decimals to be shown for each column
R> print(ttKRx,include.rownames=FALSE,
  latex.environment="center", caption.placement="top",table.placement="!h")

R> proc.time()-tic
  user  system elapsed
  0.12   0.02   0.17

```

TABLE 2. MLE results for error variance using Krinsky-Robb method

variable	estimate	s.e.	lower	upper
error variance	0.224	0.010	0.205	0.243