Definition of Environmental Economics

Pareto-optimality and market failures

A formal model of externalities
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Overarching and Related Fields

- **Environmental Economics and Economics**
  - Microeconomics (consumer, firm, markets)
  - Applied Field (draws on microeconomics and all main fields)

- **Environmental Economics and Resource Economics**
  - Environmental Economics: market failure, externalities, policy design; often static
  - Resource Economics: Production and use of natural resources; often dynamic
  - Divisions blurred; Both included in traditional grad sequence
  - FREC 5984 Special Studies: Advanced Natural Resource Economics
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Main focus of Environmental Economics:

humans ↔ nature

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effect on other humans

If impacts are ignored or remain uncompensated (or unpaid):
externality

Externalities have implications on human welfare →
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**Pareto-optimality**

A *normative* criterion to judge desirability of different economic outcomes.

Economic outcome such that a *reallocation of resources* cannot make at least one person better off without making another worse off.

Does not address issues of equity and fairness.

Pure focus on *efficiency*.

Impossible to implement in practice. Politically unsustainable.
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Welfare theorems

First welfare theorem
Complete and perfectly competitive markets will lead to Pareto-optimal allocation of resources.

Second welfare theorem
In complete and perfectly competitive markets: Any Pareto-optimal allocation can be supported by a price system arising from a corresponding redistribution of income via lump-sum taxes and transfers.
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Point of departure

Conditions for welfare theorems usually not met: markets are incomplete (fail to address externalities, free-riding, etc), and - at best - impurely competitive (monopolies, oligopolies, etc).

Think of these fundamental concepts as points of departure.

Objective of Env. Econ.:

1. Understand when and why free markets fail to deliver efficient outcomes.
2. Find ways to mitigate these “failures”
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1. Understand when and why free markets fail to deliver efficient outcomes.
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When the utility or production function of an economic agent depends on real variables generated via consumption or production by another economic agent, and no consideration is given to the resulting change in the first agent’s wellbeing.

Real variables: noise, pollution, visual impediments, etc.

So interactions that only result in price changes don’t count (i.e. restaurant suffers because new one opens across the street).
Externalities

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Example
Public goods

Goods that are (purely or at least partially) non-rival and non-exclusive.

Similar implications on welfare as externalities: Markets alone will not accomplish efficient allocation.

Many environmental problems (and solutions) share PG characteristics.

Thus, study of PG-type problems is also topic of Env.Econ.
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In a nutshell ...

Externalities, PG’s → failure of Fundamental Welfare Theorems

These failures → analytical starting point for Environmental Economics
In a nutshell ...

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At the most general level, we could also define Env. Economics as the study of the **failure of coordination**:

- missing / incomplete / impure markets
- incomplete / poorly enforced property rights
- inability of affected individuals to make collective decisions

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Utility

Setup

Pareto optimality
Competitive market
Market Intervention

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A formal model of externalities

\[ U_i = U_i \left(x_i, z_i, E\right), \quad i = 1, 2 \]

\[ \frac{\partial U \left(\cdot\right)}{\partial x} > 0 \]

\[ \frac{\partial U \left(\cdot\right)}{\partial z} > 0 \]

\[ \frac{\partial U \left(\cdot\right)}{\partial E} < 0 \]

\( x_i \): dirty good
\( z_i \): clean good
\( E \): emission (exogenous to consumers)
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Production

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\[ x = f (l_x, E) \]  
\[ \frac{\partial f (.)}{\partial l_x} > 0 \]  \( (2) \)

\[ \frac{\partial f (.)}{\partial E} > 0 \]  

\[ z = g (l_z) \]  \( (3) \)

\[ l = l_x + l_z \]  \( (4) \)

\( l, l_x, l_z: \) labor
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\( l, l_x, l_z: \) labor
Setup highlights

- **So here emissions** = input.
- Reduction in pollution reduces output of $x$.
- Could also let $x, E$ be joint products (more notational clutter)
- First task: find conditions for Pareto optimality
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Optimization problem

\[
\begin{align*}
\max_{x_1, x_2, z_1, z_2, l_x, l_z, E} & \quad \mathcal{L} = U_1(x_1, z_1, E) + \\
& \quad \lambda_u (U_2(x_2, z_2, E) - \bar{u}_2) + \\
& \quad \lambda_x (f(l_x, E) - x_1 - x_2) + \\
& \quad \lambda_z (g(l_z) - z_1 - z_2) + \\
& \quad \lambda_l (l - l_x - l_z)
\end{align*}
\]

key: Must hold agent 2 at a pre-specified utility level (her current level, for example)
Optimization problem

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\max_{x_1, x_2, z_1, z_2, l_x, l_z, E} \mathcal{L} =
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U_1(x_1, z_1, E) +
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FOCs: Consumption

\[
\frac{\partial U_1 (.)}{\partial x_1} = \lambda_x \\
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Efficiency in Consumption

\[ \frac{\partial U_1}{\partial x_1} \frac{\partial U_1}{\partial z_1} = \frac{\partial U_2}{\partial x_2} \frac{\partial U_2}{\partial z_2} = \frac{\lambda_x}{\lambda_z} \]  

(7)

MRS between goods equal for both individuals.
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Efficiency in Production

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Shadow value of MPL for each good = shadow price of labor for each good.
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Efficiency in Exchange

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\frac{\partial U_i(.)}{\partial x_i} = \frac{\partial g(.)}{\partial l_z} = \frac{\lambda_x}{\lambda_z}, \quad i = 1, 2
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Slope of Production Possibility curve = slope of each person’s indifference curve (IC)
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FOCs: Emission

\[
\frac{\partial U_1 (.)}{\partial E} + \lambda_u \frac{\partial U_2 (.)}{\partial E} + \lambda_x \frac{\partial f (.)}{\partial E} = 0 \quad (10)
\]

rewrite: divide by \(\lambda_x\), substitute (6):

\[
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Interpretation

- More E lowers utility directly, but increases it indirectly via increased production and consumption of x.
- Left hand fractions = marginal willingness to give up x for a reduction in E (or “marginal benefits of reducing E, as measured in x”).
- RHS: cost in reduced production of x from reducing E.
- Reduction in E benefits both, so marginal cost in foregone production needs to be compared to sum of marginal benefits.
- In equilibrium, sum of marginal benefits must equal marginal cost of reducing emissions.
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- left hand fractions $= \text{marginal willingness to give up } x \text{ for a reduction in } E$ (or “marginal benefits of reducing } E, \text{ as measured in } x”)
- RHS: cost in reduced production of $x$ from reducing $E$
- Reduction in $E$ benefits both, so marginal cost in foregone production needs to be compared to sum of marginal benefits.
- In equilibrium, sum of marginal benefits must equal marginal cost of reducing emissions.
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Symmetry to Public Goods case

- Some externalities (or reduction thereof) are like PGs
- Here: Reduction in E has PG characteristics
- Benefits enjoyed by one agent from less E do not interfere with benefits for the other
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- So in this light, equ. (11) is also the Lindahl-Samuelson condition for the efficient allocation of a PG
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Note:
Current solution \( \{x_1^*, x_2^*, z_1^*, z_2^*, l_x^*, l_z^*, E^*\} \) is just one of many Pareto-efficient outcomes.

Setting a different utility for individual 2 (\( \bar{u}_2 \)) would yield a different solution and optimal level of pollution.
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Can free, competitive market produce P.O. outcome?

2 Consumers, 2 firms economy.

Individuals maximize utility, firms maximize profits.

Given: prices $p_x, p_z$, wage $w$, income $y_i$, $i = 1, 2$
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Consumer problem and FOCs

\[ \max_{x_i, z_i} \mathcal{L} = U_i(x_i, z_i, E) + \lambda_i (y_i - p_x x_i - p_z z_i), \quad i = 1, 2 \] (12)

\[ \frac{\partial U_1(.)}{\partial x_1} / \frac{\partial U_1(.)}{\partial z_1} = \frac{\partial U_2(.)}{\partial x_2} / \frac{\partial U_2(.)}{\partial z_2} = \frac{p_x}{p_z} \] (13)

Marginal rate of substitution (MRS) equals price ratio for all consumers: \( \rightarrow \) efficiency in consumption
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Firm z’s problem and FOCs

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\max_{l_z} \pi_z = p_z g(l_z) - w l_z \tag{14}
\]

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p_z \frac{\partial g(.)}{\partial l_z} = w \tag{15}
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Value of marginal product of labor (MPL) equals wage rate → efficiency in production
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Firm x’s problem and FOCs

Firm x must select both labor and emission inputs to max. profits, treating emission as a free factor

\[
\max_{l_x, E} \pi_x = p_x f(l_x, E) - w l_x
\]  

(16)

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\]

(17)
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p_x \frac{\partial f(.)}{\partial E} = 0
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Efficiency in exchange is met, since from (13), (15), and (23):

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Slope of IC equals slope of production function for both consumers.
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Discussion

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Before:

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Now:

$$p_x \frac{\partial f (.)}{\partial E} = 0 \quad (20)$$
Discussion, cont’d

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Firm emits pollution up to level where the value of the marginal product of pollution = 0 (any further pollution input won’t help production)

$E$’s impact on consumers is not considered in deriving its optimal quantity.

Competitive market with externality (essentially a missing input market) fails to produce a Pareto-efficient outcome.
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Env. econ. deals with design of policies to correct market failure.

Example: Impose Pigouvian tax (Pigou, 1920) for each unit of $E$ used in production of $x$:

$$\tau^* = -p_x \left( \frac{\partial U_1(.)}{\partial E} \frac{\partial E}{\partial x_1} + \frac{\partial U_2(.)}{\partial E} \frac{\partial E}{\partial x_2} \right)$$ (21)
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Revised Firm $x$’s problem and FOCs

Emission input now has price of $\tau^*$:

$$\max_{l_x,E} \pi_x = p_x f(l_x, E) - wl_x - \tau^* E$$

(22)

$$p_x \frac{\partial f(.)}{\partial l_x} = w$$

$$p_x \frac{\partial f(.)}{\partial E} = \tau^* = -p_x \left( \frac{\partial U_1(.)}{\partial E} / \frac{\partial U_1(.)}{\partial x_1} + \frac{\partial U_2(.)}{\partial E} / \frac{\partial U_2(.)}{\partial x_2} \right)$$

(23)

Pigouvian Tax
Revised Firm $x$’s problem and FOCs

Emission input now has price of $\tau^*$:

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Pigouvian Tax
Revised Firm $x$’s problem and FOCs

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Pigouvian Tax
Figure 1.1: Efficient Pollution Level and Pigouvian Tax

\[ p_x \left[ \frac{\partial U_1}{\partial E} + \frac{\partial U_2}{\partial E} \right] = MUC_E \]

\[ -p_x \frac{\partial U_2}{\partial E} = MUC_{2E} \]

\[ p_x \frac{\partial U_1}{\partial E} = MUC_{1E} \]

\[ p_x \frac{\partial f}{\partial E} = VMP_E \]

Panel A
Graphical Analysis: Abatement

\[ \frac{\partial f}{\partial A} = MC_A \]

\[ P_x \left[ \frac{\partial U_1}{\partial A} + \frac{\partial U_2}{\partial A} \right] = MUB_A \]

\[ P_x \frac{\partial U_2}{\partial x_2} = MUB2_A \]

\[ P_x \frac{\partial U_1}{\partial x_1} = MUB1_A \]

\[ A^* = E^* - E^* \]