Consumer Surplus
Exact welfare measurement

Outline
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Consumer Surplus

Definition:

\[ C = \int_{p^0_j}^{p^1_j} x_j(p, y, q) \, dp_j \]

If \( x_j(p^1_j, p_{-j}, y, q) = 0 \), \( p^1_j \) = "choke price"

Based on ordinary ("Marshallian") demands

Easily computed from observed demand

What does it measure?

How does it relate to WTP, WTA?

Substantial literature from '70's to early '90's
In general, $C$ has no meaningful interpretation in terms of WTP, WTA.

Price change along ordinary demand curve $\rightarrow$ utility change

So area behind demand curve $\neq$ utility-constant welfare measure

Re-write $C$ (via Roy’s identity) to gain additional insight:

\[
C = \int_{p_j^0}^{p_j^1} \left( V_{p_j}(p, y, q) - \frac{V_y(p, y, q)}{\lambda(p, y, q)} \right) dp_j
\]

\[
= \int_{p_j^0}^{p_j^1} \frac{V_{p_j}(p, y, q)}{\lambda(p, y, q)} dp_j
\]

$\lambda(\cdot) = \text{marginal utility of income}$

Numerator $= \text{marginal (dis)utility of price}$
IFF $\partial \lambda(.) / \partial p_j = 0$ we can write

$$C = \frac{1}{\lambda(p-j, y, q)} \int_{p_0^j}^{p_1^j} - V_{p_j}(p, y, q) dp_j$$

$$= \frac{1}{\lambda(p-j, y, q)} (V(p^0, y, q) - V(p^1, y, q))$$

In that case, $C$ is indeed a money-metric reflection of the change in utility

Thus, it has some quantitative meaning for welfare analysis

But: How likely is $\partial \lambda(.) / \partial p_j = 0$?
Consumer Surplus: Interpretation

Roy's identity:

\[ x_j(p, y, q) = \frac{-V_{pj}(p, y, q)}{V_y(p, y, q)} \]

\[ x_j(p, y, q) \times V_y(p, y, q) = -V_{pj}(p, y, q) \]

Diff. w.r.t. y:

\[ x_j(.) \times V_{yy} + \frac{\partial x_j(.)}{\partial y} V_y(.) = -V_{pj, y}(.) \]

IFF \( \partial \lambda(.) / \partial p_j = 0 \) \( \rightarrow \) \( V_{y, p_j} = V_{p_j, y} = 0 \), we have

\[ \frac{\partial x_j(.)}{\partial y} = -\frac{x_j(.) V_{yy}(.)}{V_y(.)}, \text{ and} \]

\[ \frac{\partial x_j(.)}{\partial y} \times \frac{y}{x_j(.)} = \eta_j = -\frac{y V_{yy}(.)}{V_y(.)} \]
\[ \eta_j = \text{income elasticity of demand for good } j \]

Same for all \( j \) for which \( \frac{\partial \lambda(.)}{\partial p_j} = 0 \)

For \( C \) to have meaningful implication, must have:

\[ \frac{\partial \lambda(.)}{\partial p_j} = 0 \text{ and } \eta_j = \eta \]

for all goods whose price may change during the analysis.

Plausibility depends on context.

Often: most prices are assumed unchanged during research period

Only a few prices allowed to change

Assumption of equal income elasticity for those goods often reasonable

Example: Set of local recreation sites
Multiple price changes

From budget constraint:

\[ y = \sum_{j=1}^{J} p_j \cdot x_j (p, y, q) + z (p, y, q) \]

\[ 1 = \sum_{j=1}^{J} p_j \frac{\partial x_j (.)}{\partial y} + \frac{\partial z (.)}{\partial y} = \]

\[ \sum_{j=1}^{J} p_j \frac{\partial x_j (.)}{\partial y} \frac{yx_j}{x_j y} + \frac{\partial z (.)}{\partial y} \frac{yz}{zy} \]

\[ y = \sum_{j=1}^{J} \eta_j p_j x_j (.) + \eta z (.) \]

Now impose \( \eta_j = \eta, \forall j \):

\[ y = \eta \sum_{j=1}^{J} p_j x_j (.) + \eta z (.) \]

Implies \( \eta = \eta z = 1 \)

Unlikely, intuitively and econometrically
Uniqueness of C

Additional problem with $C$:
Generally not unique for multiple price changes

IFF income, all other prices remain constant: $C$ is unique -
same condition as needed for meaningful interpretation!

Summary

For $C$ to be unique and have a meaningful interpretation, we need:

Equality of income elasticity for all goods of interest (i.e. with a price change)

Equivalent condition:

Ordinary demand cross-price effects need to be equal
Consumer Surplus as an Approximation

Tempting to use $C$ to measure welfare effects (based on observed demand)

How far off would we be compared to $CV$, $EV$?

For a normal good, we have:

$CV < CS < EV$ for a price decrease

$EV < CS < CV$ for a price increase

Willig’s Bounds (1976)

Shows (for single good) that error will be small if:

income elasticity is small, and / or

change in $C$ is small relative to the overall budget

Then $C$ error will be trivial,...

... especially compared to other possible errors (measurement, specification, estimation)
Willig’s Bounds (1976)

\[ \frac{C_y}{\eta} \times \frac{\eta}{2} \leq \left| \frac{C-CV}{CV} \right| \leq \frac{C_y}{\eta} \times \frac{\bar{\eta}}{2} \]

\( \eta \) = lowest income elasticity in price region under consideration

\( \bar{\eta} \) = highest income elasticity in price region under consideration

Just et al. (2004) extension

Show that Willig’s bounds carry over to multiple goods / multiple price-changes case

Upshot:

In many empirical applications \( C \) will be a reasonable approximation to the true/correct welfare measure.
Hausman (1981), Hanemann (1980) show techniques to directly calculate CV, EV from ordinary demand functions.

No need to use $C$ and hope for a "decent approximation".

Fundamental insight:

Ordinary demands (if consistent with a well-defined U-max problem) contain sufficient information to recover relevant preference parameters.

Integrability conditions

Ordinary demands must satisfy integrability conditions.

Must obey curvature conditions implied by theory.

Slutsky substitution matrix (contains own-and cross-price effects) must be symmetric and negative semi-definite.

This can be enforced in the econometric model.
Two important identities:

(1) \( x_j(p, E(p, u^0, q), q) \equiv h_j(p, u^0, q) \quad \forall j \) (shown earlier)

Observed demand represents solution to both U-max and E-min problems

(2) \( \bar{u} \equiv V(p, E(p, \bar{u}, q), q) \)

Shows link between \( V \) and \( E \) for a reference level of \( \bar{u} \)

Diff. w.r.t. \( p_j \):

\[
\frac{\partial V(.)}{\partial p_j} + \frac{\partial V(.)}{\partial y} \frac{\partial E(.)}{\partial p_j} = 0
\]

Re-write and use Roy:

\[
\frac{\partial E(p, \bar{u}, q)}{\partial p_j} = -\frac{\partial V(.)}{\partial p_j} = x_j(p, y, q)
\]
Integrability conditions

\[ \frac{\partial E(p, \bar{u}, q)}{\partial p_j} = -\frac{\partial V(.)}{\partial p_j} = x_j(p, y, q) \]

Interpret as differential equation relating income and \( p_j \)

Solve to obtain \( E(.) \)

From there, get relevant welfare measure.

Example:

Specify \( x_j(.) \), get data, estimate parameters.

Solve differential equation

\[ \frac{\partial y(p_j)}{\partial p_j} = x_j(p_j, p_{-j}, y, q) \]

Obtain

\[ y(p_j, k(p_{-j}, q)) \]
Integrability conditions

\[ y(p_j, k(p_{-j}, q)) \]

\( k(.) = \) constant of integration

Combines all terms other than \( p_j \)

If \( p_{-j}, q \) remain fixed, \( y(.) = "quasi-expenditure" \) function

Use it to compute welfare measures

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Given ordinal nature of utility, we can normalize the constant of integration to the baseline utility:

\[ u^0 = k(p_{-j}, q) \]

This yields the "quasi-expenditure" function \( \hat{E}(p_j, u^0) \)

"quasi", because we can only recover preference components related to good \( j \)

Everything else in the full expenditure function assumed fixed
Limitations

This only works if the differential equation has a closed-form solution

Satisfied by only a handful of functions

Vartia (1983) introduces numerical algorithm that allows computation of welfare effects for a wide range of specifications

But what about multi-good price changes?

Multi-good case

LaFrance and Hanemann (1989) explore "quasi-" results for demand systems.

Ordinary demand system must satisfy integrability conditions

System of partial differential equations can be integrated back to yield the quasi-expenditure and quasi-indirect utility function.

Same general idea / procedure as for the single-good case.

von Haefen (JARE, 2002) shows a large collection of functional forms, integrability conditions, and results.
Applied welfare analysis for price changes is fairly straightforward.

Techniques are well-understood and widely accepted:

1. Specify properly restricted demand system
2. Estimate parameters econometrically
3. Derive quasi-IUF and / or expenditure function
4. Derive CV and / or EV